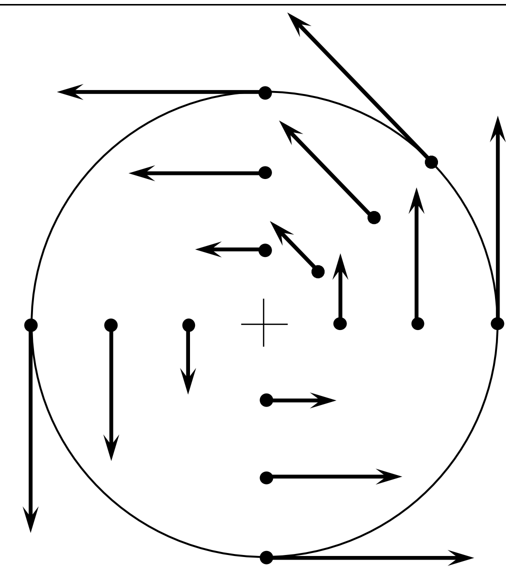
31 ROTATIONAL KINEMATICS

1. Compare and contrast circular motion and rotation? Address the following • Which involves an object and which involves a system? • Does an object/system in circular motion have any rotational characteristics? • Does a rotating object/system have any circular motion characteristics?	

2. A disk rotating through its center is shown in the diagram at the right. The instantaneous velocities for several points on the rotating disk indicated by the vector arrows, where the length of each vector indicates the speed of each point.
   (A) How is speed related to the distance from the axis of rotation?
   (B) All points that are the same distance from the axis have the same ____ , but these points all have different _____.
   (C) All points that lie on the same radius have the same _______ , but they have different _______.
   (D) Do any two points on a rotating system have the same velocity? If not explain why.



(A)	
(B)	
(C)	
(D)	

3. The disk is essentially an object/system comprised of individual points that are all in circular motion at different velocities. There is an important quantity that every point (except the point at the axis) moves through during a specific time interval. (A) What is this key quantity? (B) How is measured? (C) How is it related to revolutions	(A) (B) (C)

4. Angular Quantities	Variable and units	Converstion to tangential quantities
(A) Angular displacement		
(B) Angular velocity		
(C) Angular acceleration		

5. Important conversions (A) revolutions to radians (B) rpm to rad/s	(A)	(B)

Equation Comparison	Linear Motion	Rotation (system)	Circular Motion (point)
6. Default positive direction	Right and Up	Counterclockwise (ccw)	Counterclockwise (ccw)
7. Time	t = elapsed time	t = elapsed time	t = elapsed time T = time of ***ONE*** cycle (rev) $\boxed{T = \frac{2\pi}{\omega} = \frac{1}{f}}$
8. Distance / displacement	$\Delta x = x - x_0$	$\Delta\theta = \theta - \theta_0$	$d_{one\ cycle} = 2\pi r$ $\quad \Delta x = r\theta$
9. Constant speed / velocity	$v = \frac{d}{t}$ $\quad \vec{v} = \frac{\Delta x}{t}$	$\omega = \frac{\Delta\theta}{t}$	$v = \frac{2\pi r}{T}$ $\quad v = r\omega$
10. Acceleration	$\boxed{v_x = v_{0x} + a_x t}$ $\boxed{x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2}$ $\boxed{v_x^{\ 2} = v_{0x}^{\ \ 2} + 2a_x\left(x - x_0\right)}$	$\boxed{\omega = \omega_0 + \alpha t}$ $\boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2}$ $\omega^2 = \omega_0^{\ 2} + 2\alpha\left(\theta - \theta_0\right)$	$\boxed{a_c = \frac{v^2}{r}} = \omega^2 r$ $\quad a = r\alpha$

11. An object in uniform circular motion of radius 4.0 m completes 10 rev in 2.0 s. Determine the (A) angular displacement (B) tangential displacement (C) angular velocity (D) tangential velocity (E) angular acceleration (F) tangential acceleration	(A) Angular displacement	(B) Tangential displacement
	(C) Angular velocity	(D) Tangential velocity
	(E) Angular acceleration	(F) Tangential acceleration
12. An object at rest begins to rotate uniformly with an angular acceleration of 6.0 rad/s^2. How many revolutions has the object completed when it reaches the point where it is spinning at 120 rpm?		

13. An object with an initial angular velocity of 20 rad/s accelerates at 10 rad/s^2 for 5.0 s with a radius of 3.0 m. Determine the objects (A) angular displacement (B) radial acceleration at time t = 5.0 s	(A) Angular displacement	(B) Radial acceleration
14. An object initially at rest accelerates uniformly in a circle with radius 4.0 m for 10 s until reaching a tangential speed of 20 m/s. Determine the objects (A) final angular velocity (B) angular acceleration (C) radial acceleration at time t = 10 s (D) angular displacement	(A) Final angular velocity	(B) Angular Acceleration
	(C) Radial acceleration	(D) Angular displacement

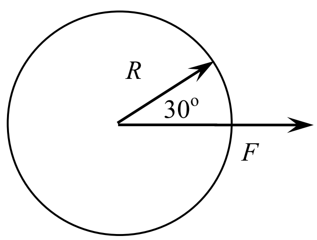
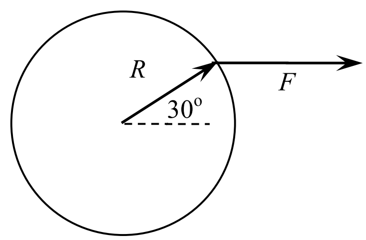
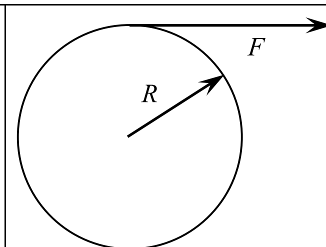
Kinematic Graphs	**Slope**	**Area**
15. $x - t$		
16. $\theta - t$		
17. $v - t$		
18. $\omega - t$		
19. $a - t$		
20. $\alpha - t$		

32 TORQUE AND CENTER OF GRAVITY

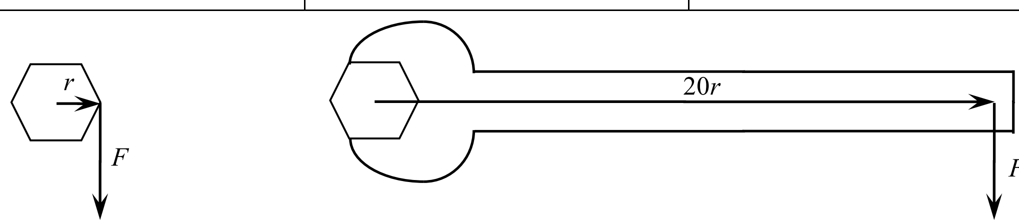
21. **Torque**: A force applied at a distance from the center of mass of an object that causes rotation.	$\vec{\tau} = \vec{r} \times \vec{F}$ $\boxed{\tau = r_{\perp}F = rF\sin\theta}$	$\vec{\tau}$	Torque: The rotational equivalent of force (N•m)
		$\vec{r}$	**Moment Arm**: The distance from the axis of rotation to the point where the force creating the torque is applied (the tail of the force vector). (m)
		$\vec{F}$	Force creating the torque (N)

22. In rotation some objects will be round having a radius, R. The length of the moment arm, r, often appears the be a radius as well. What exactly is the moment arm, and how will we distinguish it from an ordinary radius?

23. The scalar version of the equation, $\tau = rF\sin\theta$, is used to solve rotation problems. In the scenarios depicted at the right a disk with a radius $R = 5\text{ m}$ is acted upon by an applied force of 10 N. For each scenario determine the
(A) length of the moment arm, r.
(B) the angle, θ.
(C) the applied torque, τ.

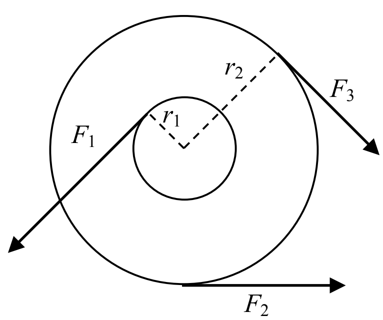
		
(A)		
(B)		
(C)		

24. Turning a bolt. What is the advantage to using a wrench when turning a bolt?



25. What is the relationship between tangential force, torque, tangential acceleration, and angular acceleration?		
26. Net Torque		
27. The sum of torque and angular acceleration is similar to the sum of linear forces and linear acceleration. Fill in the chart	What motion results when the net linear force acting on an object **is equal** to zero?	What motion results when the net torque acting on an object **is equal** to zero?
	This(these) type(s) of motion can be described with what terms?	This(these) type(s) of motion can be described with what terms?
	What motion results when the net linear force acting on an object **is not equal** to zero?	What motion results when the net torque acting on an object **is not equal** to zero?

28. A pulley with two different radii $r_1 = 1$ m and $r_2 = 3$ m is acted upon by three forces. $F_1 = 6$ N and $F_2 = 4$ N and F_3 is unknown. The moment of inertia of this pulley is $I = 6$ kg m^2



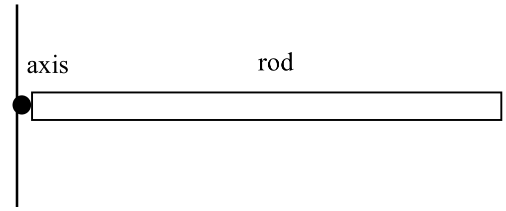
(A) Determine the magnitude of F_3 so that the pulley remains in equilibrium.

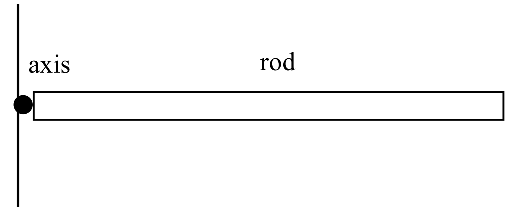
(B) Determine the magnitude of F_3 so that the net torque is 4 N•m.

(A)

(B)

29. Gravitational Torque





30. Center of Gravity

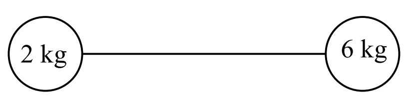
31. How can the center of gravity be found experimentally?

32. Center of Gravity

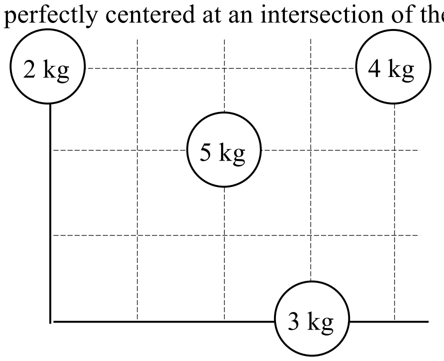
$$\boxed{r_{cm} = \frac{\Sigma mr}{\Sigma m}}$$

r_{cm}	distance from (0,0) to center of mass (m)
m	mass (kg)
r	distance from (0,0) to each mass (m)

33. Determine the center of gravity for the following arrangement. The rod connecting the masses is massless and the masses are 2 m apart.



34. Determine the center of mass for the following arrangement. Each line on the grid system shown below is separated by 1 m. Each mass is perfectly centered at an intersection of the grids.



35. Difference between center of mass and center of gravity

33 MOMENT OF INERTIA

36. **Moment of Inertia**
The inertia of a rotating object.

Similar to mass m in linear motion. In linear motion an object with more mass has more inertia. More inertia means it is more difficult to accelerate. If it is more difficult to accelerate, then it is more difficult to change its velocity (speed it up, slow it down, or change its direction).

If an mass has a greater moment (momentum) of inertia I, then it is more difficult to give it angular acceleration α. If it is more difficult to give it angular acceleration, then it is more difficult to change its angular velocity ω. As a result it is more difficult to rotate if it is standing still and more difficult to speed up, slow, or stop the objects rotation if it is already rotating.

37. For most rotating objects the moment of inertia is solved using integral calculus. In this course the moment of inertia will be given for rotating objects. The moments of inertia for some commonly encountered objects are given at the right.

Hoop	$I = MR^2$	M = mass of object R = radius of object
Disc / cylinder / pulley	$I = \frac{1}{2}MR^2$	
Solid sphere	$I = \frac{2}{5}MR^2$	
Hollow sphere	$I = \frac{2}{3}MR^2$	
Rod rotating thru center of mass	$I = \frac{1}{12}ML^2$	M = mass of rod L = length of rod
Rod rotating about one end	$I = \frac{1}{3}ML^2$	

38. For objects in circular motion (orbiting a central point) the moment of inertia can be calculated using algebra.

$$I = \sum mr^2$$

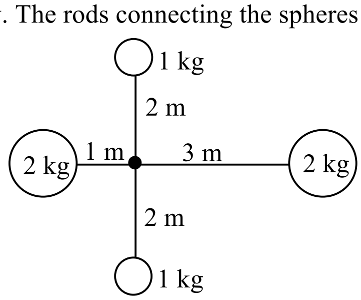
I	Moment of Inertia (kg·m^2)
m	Mass of object in circular motion (kg)
r	Radius of the circular motion (m)

39. Earth has a mass of 6.0×10^{24} kg and a radius of 6.4×10^6 m. Earth is an average distance from the sun of 1.5×10^6 m. Determine the moment of inertia of Earth's
(A) rotation
(B) revolution

(A)

(B)

40. Determine the moment of inertia of this compound body. The rods connecting the spheres are massless.

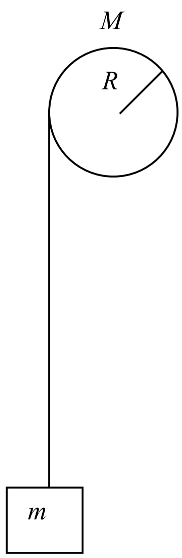


41. Net Torque	$\boxed{\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}}$ $\sum \vec{\tau} = \vec{\tau}_{net} = I\vec{\alpha}$	$\Sigma\tau$	Net torque, or sum of torque (N•m)
		I	Moment of inertia (kg•m^2)
		α	Angular acceleration (m/s^2)

42. When should you (A) sum forces? (B) sum torques? (C) sum both forces and torques?	(A)
	(B)
	(C)
43. A disk with a mass of 5 kg and a radius of 2 m is initally at rest. A 10 N tangential force is applied at the edge of the disk. Determine the (A) angular acceleration of the disk. (B) tangential acceleration of a point at the edge of the disk. (C) angular velocity of the disk at the instant it has moved through 20 revolutions. (D) tangential velocity of a point at the edge of the disk at this same moment. (E) Centripetal acceleration of this same point.	(A)
	(B)
	(C)
	(D)
	(E)

34 ROTATIONAL DYNAMICS

44. A pulley of mass $M = 3.0$ kg and radius $R = 1.0$ m has a massless string wound around it several time (like a fishing reel). A mass $m = 2.0$ kg intially at rest is suspended from one end of the string. When released it unwinds the string.



(A) Draw the FBD for the mass m and for the pulley.

(B) Key strategy

i. Sum torque for…

ii. Sum forces for…

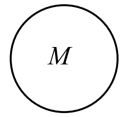
(C) Determine the systems acceleration.

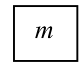
(D) Determine the tension in the sting.

(E) Determine the force exerted by the axle.

(F) Determine the angular speed of the pulley after 5.0 s.

(A)





(B) (i)

(ii)

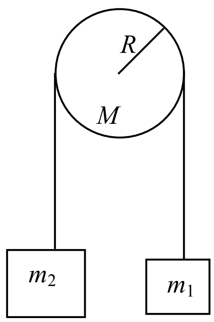
(C)

(D)

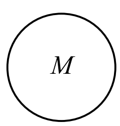
(E)

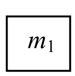
(F)

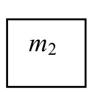
45. An Atwood machine consists of a pulley of mass $M = 4.0$ kg and radius $R = 10$ cm and two masses $m_1 = 1.0$ kg and $m_2 = 2.0$ kg as shown.



A) Draw the FBD for the pulley and each mass.







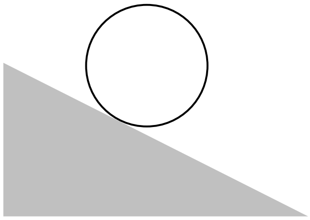
(B) Determine the acceleration of the system.

(C) Determine the tension in the string attached to mass m_2.

(D) The pulley completes 3.0 revolutions. Determine the speed of m_2.

(E) Determine the force exerted by the axle.

46. A ball of mass 2.0 kg and radius 50 cm rolls 10 m along a 30° incline.



(A) Draw the FBD for the ball if the ramp is fricitonless.

(B) Draw the FBD for the ball if the ramp is rough

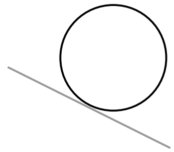
(C) Using force and kinematics determine the translational speed of the ball at the bottom of the incline **if the incline is frictionless.**

(D) Using torque, force, and kinematics determine the translational speed of the ball at the bottom of the incline **when friction is present, and the ball rolls without slipping**. The coefficients of friction are $\mu_s = 0.2$ and $\mu_k = 0.4$

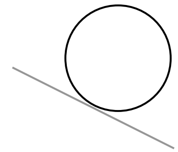
(E) Determine the friction force acting on the ball.

(F) What is the significance of the phrase "**without slipping**" ?

(A) Frictionless



(B) Rough



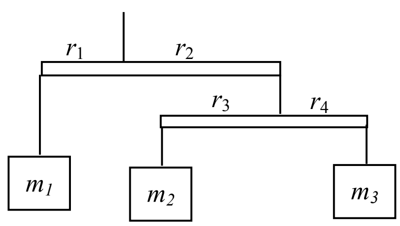
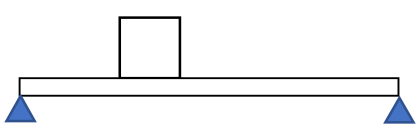
(C) Speed if incline is frictionless

(D) Speed if incline has friction and ball rolls without slipping

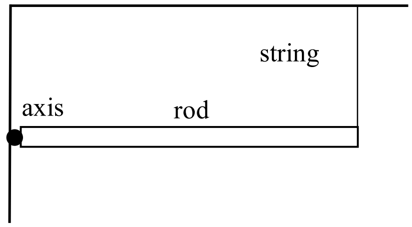
(E) Friction that allows ball to roll without slipping

(F) Significance of the phrase "without slipping"

35 STATICS

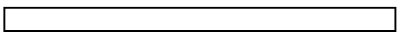
47. The key to static equilibrium problems is that (A) the sum of torque is (B) and the sum of forces is	(A)	(B)
48. A common problem is the simple tetter totter. At one end of the tetter totter a 30 kg child sits 2.0 m from the fulcrum. Determine the position of a 40 kg child on the other end that will keep the tetter totter in equilibrium.		
49. Another application is a mobile. Three masses are suspended by horizontal rods and vertical strings. m_2 = 20 g, r_1 = 10 cm, r_2 = 20 cm, r_3 = 15 cm, and r_4 = 10 cm. Determine the mass m_1 that will keep the system is equilibrium.  		
50. A 10 kg plank with a length of 3.0 m is placed between two triangular stands and a 20 kg mass is placed on the plank. The center of the mass is 1.0 m from the left edge of the plank. The apparatus is illustrated in the diagram below. Determine the support forces provided by the triangular stands. 		

51. One end of a rod is attached to a wall and able to pivot at the attachment point with the wall. A vertical string is attached to and holds up the other end of the rod. The rod has a mass of 5.0 kg and a length of 2.0 m.

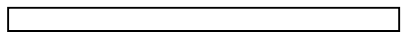


(A) Which part of the rod can be set as the pivot point in order to sum the torques acting on the rod?

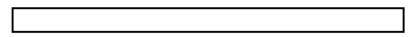
(B) Mark a pivot point at the left end of the rod, and draw the forces causing torque and their associated moment arms on the diagram of the rod below.



(C) Mark a pivot point at the left end of the rod, and draw the forces causing torque and their associated moment arms on the diagram of the rod below.



(D) Draw the FBD of all the linear forces acting on the rod



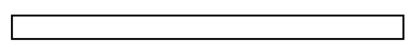
(E) Sum the torques about the left end.

(D) Sum the torques about the right end.

(E) Sum the linear forces.

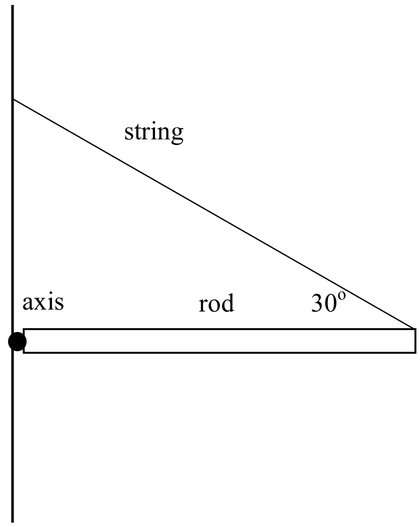
(F) **<u>The string is now cut</u>**. How does this change the problem?

(G) Draw the new FBD for the rod



(H) Determine the angular acceleration of the rod at the instant the string is cut.

(I) Can this acceleration be used to find the speed of the rod while the rod is swinging downward?

52. One end of a rod is attached to a wall and able to pivot at the attachment point with the wall. A string is attached to and holds up the other end of the rod. The rod has a mass of 5.0 kg and a length of 2.0 m. The string makes an angle of 30° with the rod.  (A) Determine the tension in the string. (B) Determine the force acting at the axis. The string is cut (C) Determine the angular acceleration of the rod at the instant the string is cut	(A) (B) (C)

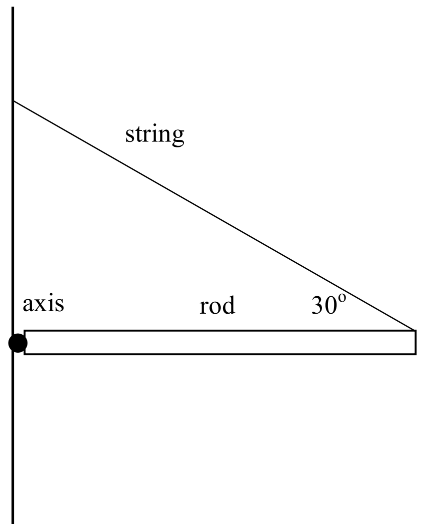
36 WORK AND ENERGY

53. Rotational Kinetic Energy	$\boxed{K = \frac{1}{2} I\omega^2}$	K_{rot}	Rotational kinetic energy (J)
		I	Moment of inertia (kg•m^2)
		ω	Angular velocity (rad/s)
54. Translational Kinetic Energy	$\boxed{K = \frac{1}{2} mv^2}$	K_{trans}	Translational kinetic energy (J)
		m	Inertia, mass (kg•m^2)
		v	Translational velocity (m/s)

55. There are now two types of kinetic energy. How can you tell when to use (A) only translational kinetic energy (B) only rotational kinetic energy (C) both translational kinetic energy and rotational kinetic energy	(A) (B) (C)

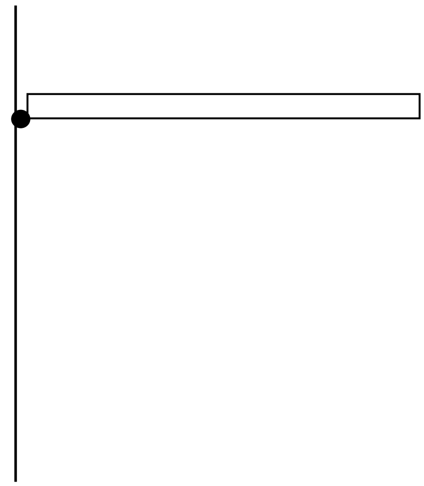
56. Determine the kinetic energy of a sphere, $m = 3.0$ kg and $r = 2.0$ m, rotating at a constant 30 rpm.			
57. A 1.0 kg ball with rolling along a surface with a speed of 4.0 m/s. Determine the ball's kinetic energy.			
58. Work	$\boxed{W = \Delta E}$	W	Work (J)
		ΔE	Change in energy (J)
59. Work kinetic energy theorem	$W_{net} = \Delta K$	W_{net}	Net work (J)
		ΔK	Change in kinetic energy (J)
60. A sphere with a mass of 3.0 kg and radius of 2.0 m is initially rotating at 30 rpm's. How much work is done on the sphere if its rotational frequency is increased to 50 rpm's.			
61. A 1.0 kg ball with a radius of 0.2 m is rolling along a surface with a speed of 4.0 m/s. The ball is accelerated to a speed of 10 m/s. Determine the work done on the ball during the acceleration.			

62. One end of a rod is attached to a wall and able to pivot at the attachment point with the wall. A string is attached to and holds up the other end of the rod. The rod has a mass of 5.0 kg and a length of 2.0 m. The string makes an angle of 30^{o} with the rod. The string is then cut.

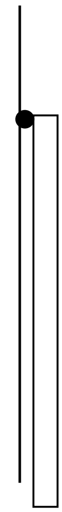


Determine the speed of a point at the end of the rod at the instant it strikes the wall.

Initial conditions (at instant string is cut)



Final Conditions (rod strikes wall)



63. Why is force and kinematics not a valid way to solve for the speed of the rod in this type of problem?

64. A ball of mass 2 kg and radius 50 cm is placed on a 10 m long a 30^{o} incline. (A) Using conservation of energy determine the translational speed of the ball at the bottom of incline if the inlcine is frictionless. (B) Using conservation of energy determine the translational speed of the ball if it is rolling down the incline without slipping.	(A) (B)
65. Friction in rolling problems is very unusual. Explain friction's role and whether or not there is an energy loss due to the work of friction in each of the following cases. (A) No friction (B) No slipping (C) Slipping	(A) (B) (C)

37 ANGULAR MOMENTUM

66. Angular Momentum	$\boxed{L = I\omega}$	L	Angular momentum ($kg \cdot m^2/s$)	ΔL	Change in angular momentum ($kg \cdot m^2/s$)
	$L = r_\perp mv$	I	Moment of inertia ($kg \cdot m^2$)	τ	Applied torque (m·N)
	$\boxed{\Delta L = \tau \Delta t}$	ω	Angular velocity (rad/s)	Δt	Elapsed time (s)
		r	Distance from axis of rotation (m)		
		m	Object mass (kg)		
		v	Tangential velocity (m/s)		

67. Determine the angular momentum for a sphere of mass 3.0 kg and radius 1.0 m that is rotating at 20 rad/s.	
68. A 20 kg child is riding 4.0 m from the axis of rotation of a merry go round. The merry go round completes 10 rotions in 40 s. Determine the angular momentum of the child.	
69. A torque of 20 m·N is applied to a rotating disk for 30 s. Determine the change in angular momentum.	

70. Conservation of angular momentum can be solved several ways. Explain when to use each of the following versions of the conservation of angular momentum equation.	$I_0\omega_0 = I\omega$	$r_0 mv_0 = rmv$
	$I_0\omega_0 = rmv$	$r_0 mv_0 = I\omega$

71. Two 5.0 kg masses are located at the ends of a 2.0 m long bar of negligible mass rotating at 4.0 rad/s about the center of the bar. The length of the bar decreases to 1.0 m. Determine the new speed of the apparatus.	
72. A 1000 kg satellite is in an elipitcal orbit about Earth. At its closest approach to Earth it is a distance of 1.5×10^7 m from the center of Earth. At this instant the satellite has a speed of 5000 m/s. Six months later the satellite reaches its greatest distance from Earth, 2.0×10^7 m from the center of Earth. Determine the satellites speed at this instant.	
73. A rod of mass 1.0 kg and with a length of 1.0 m is rotated through an axis at the end of the rod. The rod is swung horizontally with an angular speed of 6.0 rad/s. It comes into contact with a stationary sphere, mass 0.10 kg. The sphere contacts the rod at a point 0.20 m from the end of the rod. After the collision the rod continues to rotate forward with an angular speed of 3.0 rad/s. What is the velocity of the sphere as a result of the collision?	
74. What is conserved in collisions?	
75. What does conservation of linear momentum solve for?	
76. What does conservation of angular momentum solve for	