21 Mechanical Energies

1. Object / System			
2. Environment			
3. Energy (A) Definition (B) Characteristics (C) Units	(A) Energy is difficult to define. Think of energy as a characteristic of a system that allows it to move. The object can be in motion (kinetic), or the object can be in a position where releasing it allows it to move (potential). (B) (C)		
4. Kinetic energy			
5. Potential energy			
6. Mechanical Energies	Kinetic Energy	Gravitational Potential Energy	Elastic Potential Energy
(A) Equation	$K = \frac{1}{2}mv^2$	$U_g = mgh$	$U_s = \frac{1}{2}kx^2$
(B) Variable	m = mass (kg) v = velocity (m/s)	m = mass (kg) g = accel. of gravity (m/s^2) h = height (m)	k = spring constant(N/m) x = stretch (m)
(C) To possess this energy an object must be/have…			
7. Total Mechanical Energy (A) Define (B) Equation	(A) (B)		

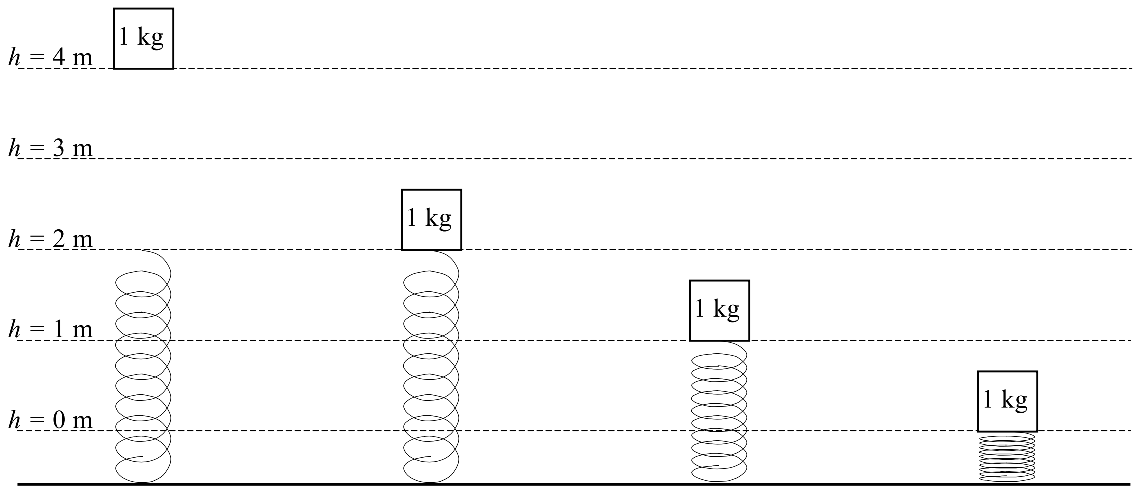
8. Conservation of Total Mechanical Energy
   (A) Define
   (B) Equation
   (C) Which energy is conserved?

(A)

(B)

(C)

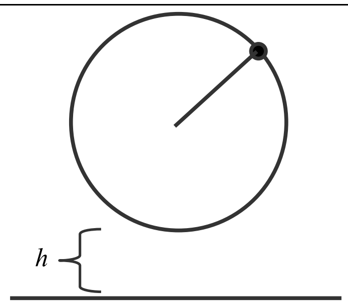
9. A 1.0 kg mass, initially at rest, is dropped from a point that is 2.0 m above a spring (k = 20 N/m). The block collides with the spring compressing the spring a maximum distance. In the diagrams at the right four instantaneous snapshots of the motion have been depicted. Determine the following at each location.



Kinetic energy				
Gravitational potential energy				
Elastic potential energy				
Total mechanical energy				

10. A projectile is launched with a speed of 30 m/s at an angle of 60°. It lands on a hill that is 40 m tall. How fast is the projectile going when it hits the ground?

11.



A 2.0 kg mass is attached to a 80 cm long string and is swung counterclockwise in a vertical circle. The bottom of the circle is h = 40 cm above the ground.

(A) Determine the minimum speed that the ball must have at the top of the loop in order to complete an entire loop.

(B) Determine the speed at the bottom of the loop.

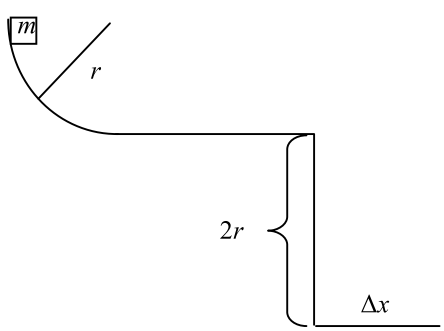
(C) The string breaks when the string reaches the bottom of the loop. Determine the horizontal displacement of the ball from when it breaks to when it hits the ground.

(A)

(B)

(C)

12.



A mass m starts from rest at the top of a smooth, frictionless, quarter circle of radius r. The upper horizontal surface is a distance $2r$ above the lower horizontal surface.
Answer all parts in terms of r, and g.

(A) Determine the speed of the mass when it reaches the bottom of the quarter circle.

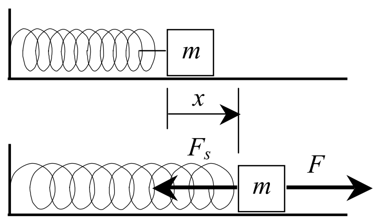
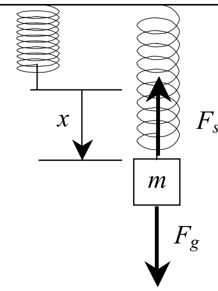
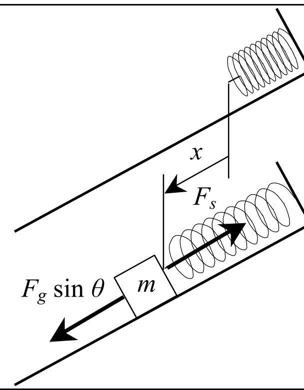
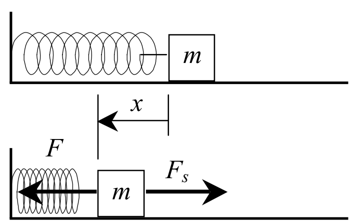
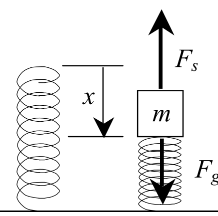
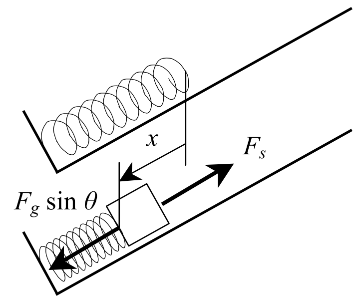
(B) Determine the horizontal distance Δx traveled by the blocks before striking the ground.

(A)

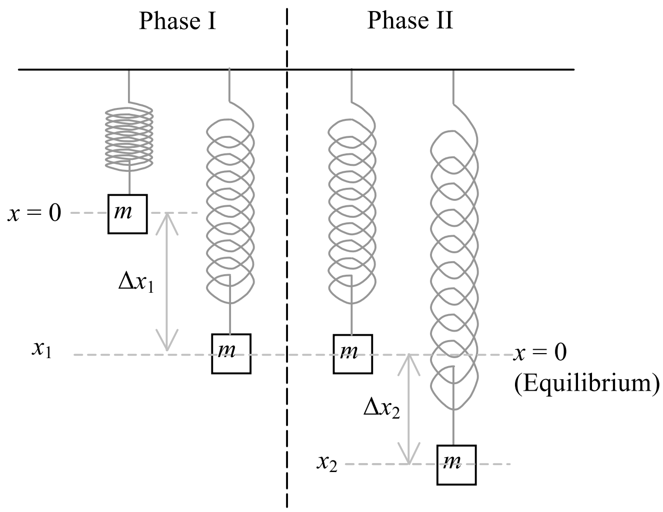
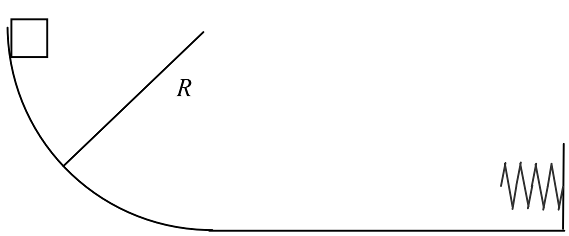
(B)

22 RESTORING FORCE AND ELASTIC POTENTIAL ENERGY

13. When a spring is stretched or compressed, what two things occur in the spring	• •

Stretched springs			
Compressed springs			

14. Hooke's Law $F_s = -kx$ (A) Why is this force known as a "**Restoring Force**?" (B) What is the significance of the minus sign? (C) What is k? (D) When can Hooke's Law be used (for what objects/systems)?	(A)
	(B)
	(C)
	(D)
15. Elastic Potential Energy $U_s = \frac{1}{2}kx^2$	

16.  A 100 g mass is attached to a spring and is lowered 25 cm until it reaches equilibrium. (A) Equilibrium is a key phrase. What is its significance? (B) Determine the spring constant k. (C) Determine the potential energy **stored in the spring**. (D) Determine the potential energy **stored in the spring / mass system**. The spring is stretched an additional 10 cm. (E) Determine the energy **stored in the spring / mass system**.	(A) (B) (C) (D) (E)
17. A toy gun consists of a spring in a tube that is used to launch a ball horizontally. The spring has a spring constant of 40 N/m and is compressed 20 cm. The ball has a mass of 0.10 kg and is initially at rest. Determine the speed of the ball when it exits the tube.	
18. A 2.0 kg mass is released from rest and moves down the frictionless surface shown below. The quarter circle section has a radius of 3.0 m. At the end of the horizontal surface the mass collides with a spring compressing the spring 40 cm. Determine the spring constant. 	

23 WORK

19.

WORK AND ENERGY

Work Kinetic Energy Theorem: $W = \Delta K$

Changes in kinetic energy are caused by changes in potential energy. Example: when a ball is dropped the change in height causes a change in potential energy. Since energy is conserved this causes an equal change in kinetic energy resulting in a change in speed. As a result, the Work Kinetic Energy Theorem can be expanded to include potential energy.

$$W = \Delta K = \Delta U$$

However, the sign on these variables is important.

Sign convention: Since work is defined as a change in kinetic energy, the sign on work is the same as the sign on the change in kinetic energy.

- If speed increases, kinetic energy increases: $+W = +\Delta K$
- If speed decreases, kinetic energy decreases: $-W = -\Delta K$

What about potential energy? In order for energy to be conserved increases in kinetic energy must be offset by decreases in potential energy, and vice versa. Example: When a ball falls the speed and kinetic energy increase, but height and potential energy decrease. The numerical values of work, change in kinetic energy, and change in potential energy are all equal. However, the sign on change in potential energy is the opposite of work.

$$\pm W = \pm \Delta K = \mp \Delta U$$

Or, even $W = \Delta E$, where care must be taken with signs.

Currently we know three mechanical energies. The above general formula can be used to generate energy specific formulas by modifying the mechanical energy formulas

Energy equations can be changed into work equations by enclosing the equation in parenthesis and then precede it with a delta symbol. For potential energies include a minus sign.

$$K = \frac{1}{2}mv^2 \longrightarrow W = \Delta\left(\frac{1}{2}mv^2\right) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$U_g = mgh \longrightarrow W_g = -\Delta\left(mgh\right) = -\left(mgh - mgh_0\right)$$

$$U_s = \frac{1}{2}kx^2 \longrightarrow W_s = -\Delta\left(\frac{1}{2}kx^2\right) = -\left(\frac{1}{2}kx^2 - \frac{1}{2}kx_0^2\right)$$

Energy is an instantaneous value associated with a specific speed, height, or compression/stretch. Work involves motion, and therefore a change in speed, height, or compression/stretch.

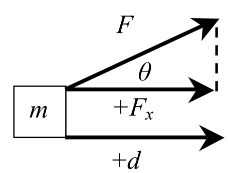
WORK AND FORCE

A force thru a distance: $W = F_{\parallel}d = Fd\cos\theta$

The components of the force and the displacement vectors that are parallel to each other are used to solve for work (cos θ adjusts the either force or displacement so that the scalars quantities representing the vectors are parallel. Distance d can be replaced with Δx, Δy, or Δh.

$$W = F_{\parallel}d$$

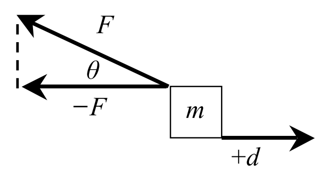
Sign convention: The direction of motion is set as the positive direction in force problems. The sign on force depends on whether force points in the direction of motion or against motion.



$W = F\ d\cos\theta$

$W = F\cos\theta\ d$

$W = +F_x\ d$



$W = F\ d\cos\theta$

$W = F\cos\theta\ d$

$W = -F_x\ d$

$$W = \pm F_{\parallel}\ d$$

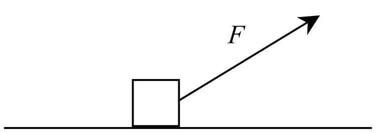
However, if force lies on an axis and displacement is diagonal (such as motion on an incline), then find the component of displacement parallel to force $W = \pm F\,d_{\parallel}$.

The work of any force can be calculated. Determine the component of force that is parallel to displacement or the component of displacement that is parallel to the force.

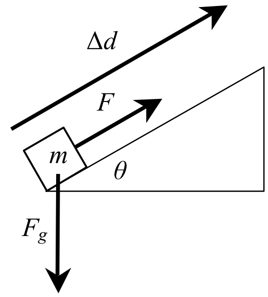
$+W_g = \left(-F_g\right)\left(-\Delta h\right)$	Moving downward
$-W_g = \left(-F_g\right)\left(+\Delta h\right)$	Moving upward
$-W_f = \left(-f\right)\left(+\Delta x\right)$	Work of friction opposes motion
$\pm W_{net} = \left(\pm\Sigma F\right)\left(+\Delta r\right)$	Net work needs the net force, ΣF

$F_{\parallel}$ is the same force with the same sign that would be used in a sum of forces equation.

When $F_{\parallel}$ has the same direction as d, then work is positive. When $F_{\parallel}$ and d have opposite directions, then work is negative.

20. A 2.0 kg mass is moving at 20 m/s when it is acted upon by a force in the direction of motion. The force acts over a distance of 30 m, and it increases the speed of the mass to a final speed of 40 m/s. Determine the work done during the period that the force acted (A) using $W = \mathbf{F} \cdot \Delta\mathbf{r}$ (B) Now try Work Kinetic Energy Theorem	(A)	
	(B)	
21.  A 30 N force, F, is applied at an angle of 37° above the horizontal to a 10 kg mass, as shown above. The mass moves 5.0 m at a constant velocity along a rough horizontal surface. (A) Determine the work done by the applied force, F. (B) Determine the work done by friction. (C) Determine the work done by gravity. (D) Determine the work done by the normal force. (E) Determine the net work.	(A)	
	(B)	
	(C)	
	(D)	
	(E)	

22.



A force, F, is applied to a 3.0 kg mass to push the mass a distance, $\Delta d = 4.0$ m, up a 30^o frictionles incline at constant velocity.

(A) Determine the change in height, Δh.

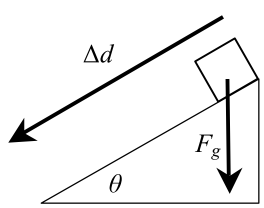
(B) Determine the change in potential energy of the mass, ΔU.

(C) Determine work of gravity, W_g, done by the force of gravity, F_g.

(D) Determine the work, W, done by the applied force, F.

(E) Determine the net work, W_{net}.

The 3.0 kg mass is now released from rest and slides 4.0 m along the frictionless 30^o incline.



(F) Determine the change potential energy.

(G) Determine the work done by gravity.

(H) Determine the net work.

(A)

(B)

(C)

(D)

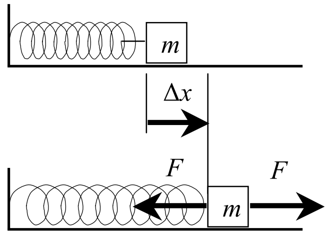
(E)

(F)

(G)

(H)

23.



A 3.0 kg mass is attached to a spring, with a spring constant of 200 N/m. A force, F , is applied to the mass causing it to move to the right while stretching the spring 20 cm.

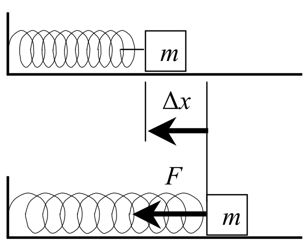
(A) Determine the change in potential energy, ΔU , of the spring/mass, system.

(B) Determine the work of the spring, W_s , due to the restoring force, F_s .

(C) Determine the work, W , done by the applied force, F .

(D) Determine the net work done while moving the mass 20 cm to the right.

The spring/mass system is released from rest.



(E) Determine the change in potential energy, ΔU , of the spring/mass, system when the mass has moved 20 cm.

(F) Determine the work, W_s , done by the restoring force, F_s , when the mass has moved 20 cm.

(G) Determine the net work done on the mass during the 20 cm motion.

(A)

(B)

(C)

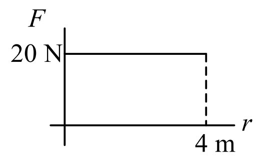
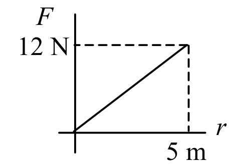
(D)

(E)

(F)

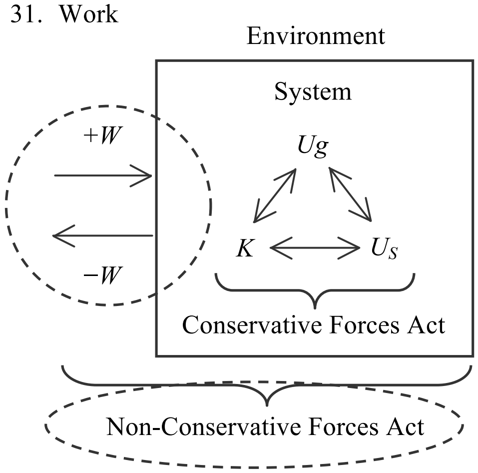
(G)

24. A 2.0 kg object slides 5.0 m along a rough surface, $\mu_k = 0.5$. (A) Determine the work of friction if the surface is horizontal. (B) Determine the work of friction if the surface is inclined at 30°.	(A) (B)
25. List situations that result in **zero work being done by a specific force**.	
26. List situations that result in **zero net work**.	
27. Which force should be used in the formula $W = F_{\parallel}d$ to calculate the same value for work that would be obtained using Work Kinetic Energy theorem, $W = \Delta K$?	
28. While the magnitude of work seems straight forward enough, the sign on work seems probelmatic. What is the best way to find the sign? (A) for individual works? (B) for net work?	

	WORK OF A CONSTANT FORCE	WORK OF A VARIABLE FORCE
29. Graphing work		
(A) Determine work		
(B) Best example of this force?		
(C) How does this relate to the work equation for each force?		
30. Putting it all together	$W = \Delta E = F_{\parallel}\Delta r = Area_{F\ vs.\ \Delta r}$	

24 WORK AND CONSERVATION OF ENERGY

31. Work



(A) State the key difference between the work of conservative and non-conservative forces.

(B) Give an example of each.

(C) Does the path taken by an object matter?

(D) Is conservation of energy violated when non-conservative forces act? Explain.

(A)	
(B) Example conservative forces	Non-conservative forces
(C) Path, conservative forces	Non-conservative forces
(C)	
(D)	

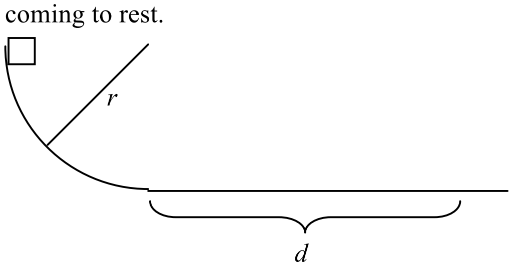
32. Look at the situation at the very beginning of the problem and access if you see a mass able to move a vertical distance h, a spring that is in a position to stretch or compress a distance x, and/or a mass that is moving v. Sum these energies at the beginning of the problem. If there are none write a zero. Do the same at the end of the problem. Examine the path from start to finish. If you see friction $-W_f$ or a motor $+W$ then apply that to the initial energies. If not leave work as a zero. If you learn new energies or different forms of work add them to the list.

$\sum E_0$	$\pm$ W	$=$ $\sum E$
0	0	0
mgh_0	$\pm F_{\parallel} d$	mgh
$\frac{1}{2}kx_0^2$	Friction horizontal $-\mu_k mg \bullet d$	$\frac{1}{2}kx^2$
$\frac{1}{2}mv_0^2$	Friction on inclines $-\mu_k mg\cos\theta \bullet d$	$\frac{1}{2}mv^2$

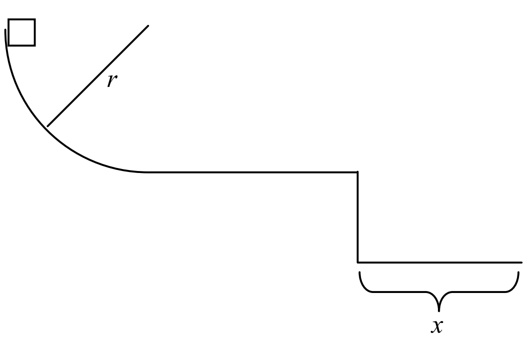
33. A mass is sliding to a stop on a rough surface. Determine the distance that the object moves before coming to a stop.

34. A mass initially at rest slides down a rough incline. Determine the speed of the object after sliding x meters along the incline.

35. A 2.0 kg mass starts from rest at the top of a smooth, frictionless, quarter circle slope of radius $r = 3.0$ m. When the mass reaches the bottom of the curve it moves along a horizontal section of track that is rough, $\mu_k = 0.50$. Determine the horizontal displacement, d, of the mass before coming to rest.



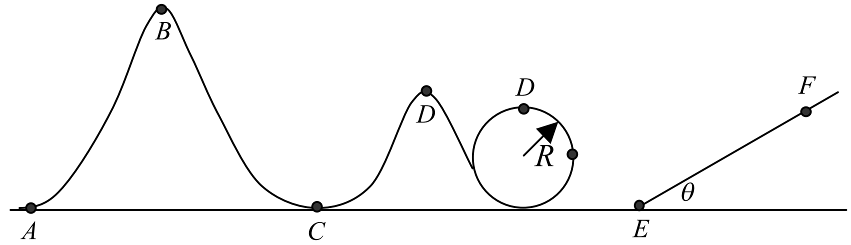
36. A 0.5 kg mass starts from rest at the top of a smooth, frictionless, quarter circle slope of radius $r = 1.5$ m. When it reaches the bottom of the curve the mass travels 2.0 m on a rough horizontal section, $\mu = 0.35$. At the end of the track there is a 0.80 m drop. Determine the horizontal displacement, x, during the drop



25 POWER	
• Define power. • Derive equations for power, and state possible units for power. • Examine the relationships between power, work, force, and time	⇒ Solve example problems in this section ⇒ Complete assignment 130 in the problem set.

37. Define power	
38. Equations for power	
39. Possible units of power	
40. Exams rarely explicitly say "solve for power." What are some likely wordings for problems that solve for power?	
41. What strategy should be used to solver for power?	
42. A $kW \cdot hr$ is the unit for what variable?	
43. A 4.0 kg mass is moved 20 m in 30 s by a 60 N force. **Determine the rate that work is being done**.	
44. A 2.0 kg mass is lifted 10 m in 5.0 s. **Determine the rate that work is being done**.	
45. A 10 kg block slides 20 meters along a rough surface, that has a coefficient of friction $\mu = 0.3$, in 30 s. **Determine the rate that heat is produced.**	
46. A force of 50 N is applied while moving a mass that is moving at 30 m/s. **Determine the rate that energy is supplied to the system.**	

47. This problem does not use realistic values. The values were chosen for ease in calculations. A roller coaster moves along a track that is frictionless with the exception of the inclined ramp starting at point *G*. The 2.0 kg roller coaster car begins at rest at, point *A*. A chain driven by a motor moves the roller coaster from point *A* to point *B* in a time $t = 10$ s. When the roller coaster arrives at point *B* it has a speed of 10 m/s. The roller coaster negogiates the track to point *H* the maximum distance up the incline. Along the incline a friction rail comes into contact with the body of the roller coaster and slows the roller coaster as it moves from *G* to *H*.



Use $g = 10$ m/s^2, $h_A = 0$ m, $h_B = 20$ m, $h_D = 12$ m, $R = 5.0$ m, $\theta = 30^o$. For the friction rail on the incline $\mu_s, = 0.30$ & $\mu_k, = 0.20$. (Note: this problem is unrealistic. The data was picked to make the calculations faster to do in class.)

(A) Determine the total mechanical energy of the rollercoaster at point *B*.

(B) Determine the work required to move from point *A* to *B*.

(C) Determine the rate that work was done on the system in moving from *A* to *B*.

(D) Determine the total mechanical energy at C.

(E) Determine the speed at *F*.

(F) Determine the maximum distance that the mass moves up the incline, as measured along the incline.

(A)

(B)

(C)

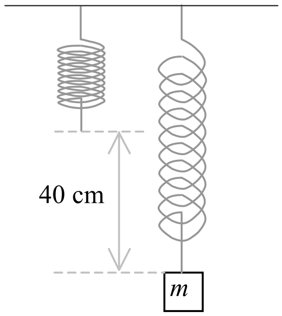
(D)

(E)

(F)

26 ENERGY IN OSCILLATIONS

48.

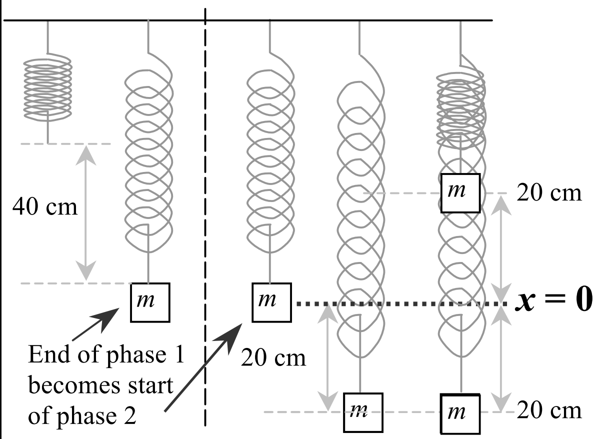


A 10 kg mass is hung from a spring and lowered to equilibrium displacing the mass 40 cm.

(A) Draw the FBD for the mass.

(B) Determine the spring constant.

It now becomes useful to set the current location as $x = 0$ (even though the spring is stretched).



The mass is then displaced an additional 20 cm from equilibrium and the mass is released.

(C) Determine the total energy in the system.

(D) Determine the speed when the mass is 10 cm from the equilibrium position.

(E) Determine the maximum speed that the spring reaches during the oscillation.

(F) Determine the period of the oscillation.

(G) Complete the chart at the bottom right, next to the diagram, indicating when each quantity is either maximum or zero.

(A)

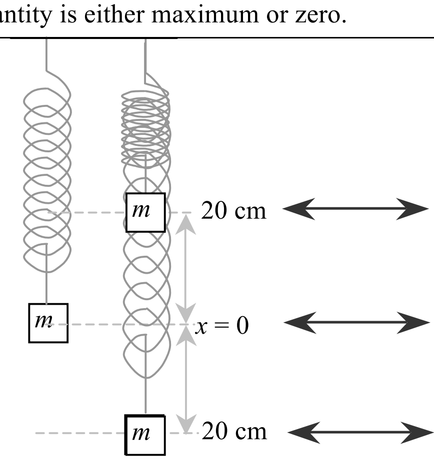
(B)

(C)

(D)

(E)

(F)

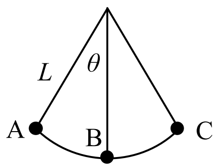


(H)

x	U_s	v	K	ΣF	a

49. Determining the change in height of a pendulum	
50. The sum of force of a pendulum (A) When displaced (B) At equilibrium	

51.



A mass of 5.0 kg hangs from a 1.0 m rope. The mass is displaced from point B to point A in the diagram below. At point A the angle that the string makes with the vertical is $\theta = 37^{o}$.

(A) Determine Δh between points A and B.

(B) Determine the energy stored in the pendulum at point A .

The pendulum is released from rest at point A .

(C) Determine the speed when the pendulum has a height of 0.10 m.

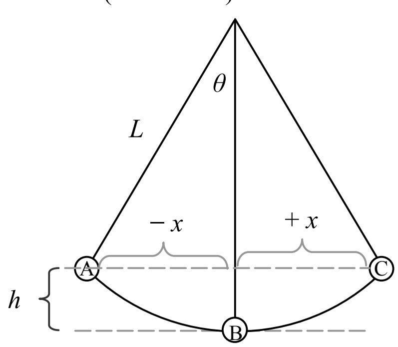
(D) Determine the maximum speed reached by the pendulum.

(E) Complete the chart, below the diagram, indicating when the quantities are either maximum or zero.

(F) Complete the potential energy graph, kinetic energy graph, and total energy graph.

(G) Are these graphs restricted to pendulums only? If not then what motions do these graphs depict?

(H) Why are force and acceleration maximum when speed is zero, and zero when speed is maximum? (Hint: FBD)



(E)	A	B	C
x & h			
U_s & U_g			
v			
K			
ΣF			
a			

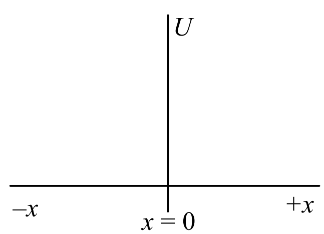
(A)

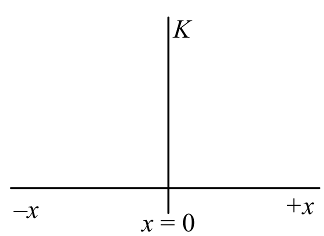
(B)

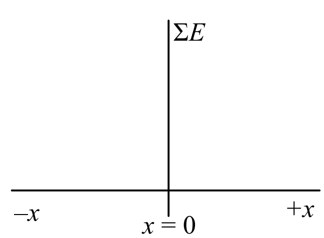
(C)

(D)

(F)



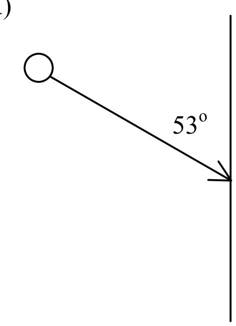
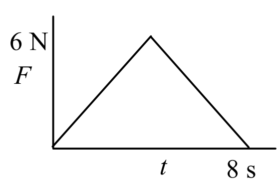




(G)

(H)

27 LINEAR MOMENTUM AND IMPULSE

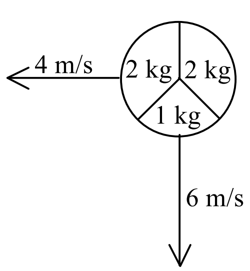
52. Momentum: Measure of how difficult it is to stop a moving object. Units: kg•m/s	$\boxed{p = mv}$
53. **Impulse**: Change in momentum. Units: kg•m/s	$\boxed{J = F\Delta t = \Delta p}$ $J = \Delta p = F\Delta t = \text{Area}_{F-t}$ $J = m\Delta v = F\Delta t = \text{Area}_{F-t}$ $J = m\left(v - v_0\right) = F\Delta t = \text{Area}_{F-t}$
54. A 5 kg mass initially at 4 m/s speeds up to 7 m/s. What is its impulse?	
55. A 6 kg mass initially at 3 m/s bounces off a wall. It loses no energy in the collision with the wall, and bounces back at 3 m/s. What is its impulse?	
56. A 4 kg mass initially at 5 m/s strikes a wall. The collision with the wall happens at an angle of 53°, as measured from the wall. No energy is lost in the collision. (A) Draw an arrow in the diagram to show the path of the mass after striking the wall. (B) Determine the impulse of the mass.	(A)  (B)
57. Determine the final velocity for a 4.0 kg mass originally moving at 2 m/s and experiencing the force graphed. 	
58. Determine the average force that must be applied for 3.0 s to accelerate a 5.0 kg mass from rest to 20 m/s.	
59. Rearranging the impulse formula yields a famous formula. Which famous formula?	
60. Total momentum: Often momentum problems involve more than one moving object.	$\Sigma p = m_1 v_1 + m_2 v_2 + \ldots$

28	CONSERVATION OF LINEAR MOMENTUM I
61. **Conservation of Linear Momentum**: The total momentum at the start of a problem must equal the total momentum at the end.	$\Sigma p_0 = \Sigma p$ $m_1 v_{1_0} + m_2 v_{2_0} = m_1 v_1 + m_2 v_2$
62. **Elastic Collisions** (perfectly elastic collisions, totally elastic collisions) are collisions where…	
63. Some examples of these collisions include…	
64. **Inelastic Collisions** are collisions where…	
65. **Kinetic Energy Lost** (Dissipated) (A) How is kinetic energy lost in a collision? (B) Where does the lost energy go and what form of energy does it become?	$\frac{1}{2} m_1 v_{1_0}^2 + \frac{1}{2} m_2 v_{2_0}^2 - K_{lost} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ (A) (B)
66. A mass $m_1 = 2$ kg moving at 3 m/s to the right collides inelastically with a mass $m_2 = 1$ kg that is moving at 6 m/s to the left. After the collision mass m_1 is moving at 2 m/s to the left. (A) Determine the total momentum (B) Determine the speed of m_2 after the collision. (C) Determine the Kinetic Energy Lost during the collision.	(A) (B) (C)

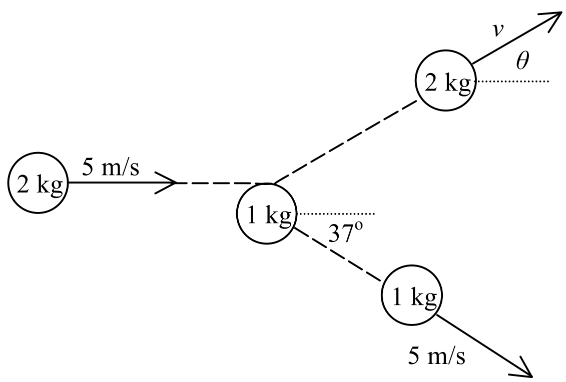
67. In **perfectly inelastic collisions** the colliding objects stick together and kinetic energy is lost.	$m_1 v_{1_0} + m_2 v_{2_0} = (m_1 + m_2)v$ $\frac{1}{2}m_1 v_{1_0}^2 + \frac{1}{2}m_2 v_{2_0}^2 - K_{lost} = \frac{1}{2}(m_1 + m_2)v^2$
68. A mass $m_1 = 1$ kg moving at 3 m/s to the right collides perfectly inelastically with a mass $m_2 = 4$ kg moving at 2 m/s to the left. (A) Determine the speed of the system after the collision. (B) Determine the kinetic energy lost during the collision.	(A) (B)
69. **Explosion**: Opposite of a perfectly elastic collision. Mass starts as one object and then splits into 2 or more objects.	$(m_1 + m_2)v_0 = m_1 v_1 + m_2 v_2$ When $v_0 = 0$ $0 = m_1 v_1 + m_2(-v_2)$ One mass moves right (+) and the other left (−) $m_1 v_1 = m_2 v_2$ The minus disappears when you rearrange the expression.
70. A 4 kg mass explodes into two pieces. A 1 kg piece moves to the right at 8 m/s. What is the speed and direction of the other piece?	

29 CONSERVATION OF LINEAR MOMENTUM II

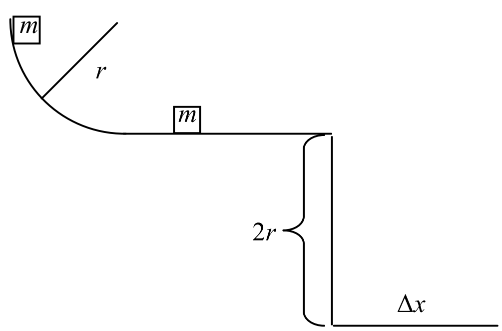
71. A 5 kg mass explodes into three pieces. A 1 kg fragment moves in the $-y$ direction at 6 m/s. A 2 kg fragment moves in the $-x$ direction at 4 m/s. Determine the speed and direction of the third fragment.



72. A 2 kg ball moving at 5 m/s in the $+x$ direction strikes a 1 kg ball at rest. The collision is slightly off center. After the collision the 1 kg ball moves with a speed of 5 m/s at an angle of 37° below the x axis. What is the speed and direction of the 2 kg ball?



73. A mass m starts from rest at the top of a smooth, frictionless, quarter circle of radius r. When it reaches the bottom of the curve it collides with a second mass m that is stationary. The collision is perfectly inelastic. The two masses move to the right until they slide off of the end of the upper horizontal surface. The upper horizontal surface is a distance $2r$ above the lower horizontal surface.



Answer all parts in terms of m, r, and g.

(A) Determine the speed of the first mass when it reaches the bottom of the quarter circle.

(B) Determine the speed of the masses after the collision.

(C) Determine the kinetic energy lost during the collision.

(D) Determine the horizontal distance Δx traveled by the blocks before striking the ground.

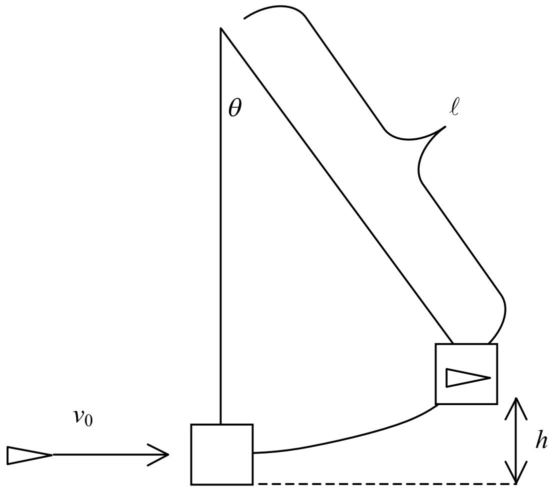
(A)

(B)

(C)

(C)

74.



In a **Ballistic Pendulum** a bullet of mass $m = 50$ g is fired into a block of mass $M = 0.5$ kg suspended by a string of length $\ell = 1.0$ m. The collision with the block is perfectly inelastic. After the collision the combined bullet and block swing together as a pendulum through an angle $\theta = 37^{\circ}$

(A) Determine the change in height Δh of the swinging pendulum.

(B) Determine the speed of the bullet and block at the bottom of the swing?

(C) Determine the speed of the bullet.

(D) Determine the kinetic energy lost in the collision.

(A)

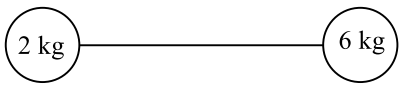
(B)

(C)

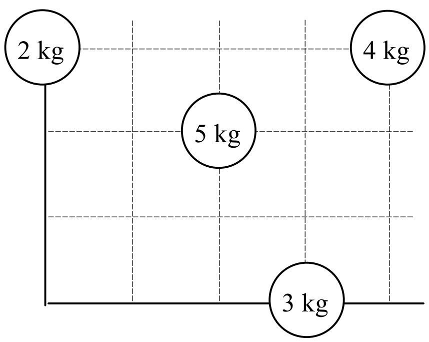
(D)

30 CENTER OF MASS			
75. Center of mass	$r_{cm} = \frac{\Sigma mr}{\Sigma m}$	r_{cm}	distance from (0,0) to center of mass (m)
		m	mass (kg)
		r	distance from (0,0) to each mass (m)

76. Determine the center of mass for the following arrangement. The rod connecting the masses is massless and the masses are 2 m apart.



77. Determine the center of mass for the following arrangement. Each line on the grid system shown below is separated by 1 m. Each mass is perfectly centered at an intersection of the grids.



78. Determine the center of mass for the following arrangement. The gray squares represent sheets of a flat material with uniform density, such as a metal. TheEach line on the grid system shown below is separated by 1 m. The mass of each square is not known, but is it really needed?